



## MATHEMATICS METHODS Year 11

### Section One: Calculator-free

Your name Solutions

Teacher name \_\_\_\_\_

#### Time and marks available for this section

Reading time before commencing work: 3 minutes  
Working time for this section: 30 minutes  
Marks available: 29 marks

#### Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet

##### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

#### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Instructions to candidates**

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet using blue/black pen. Do not use erasable or gel pen.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

## Question 1

(6 marks)

A trigonometric function is given as  $f(x) = 5 - 3 \cos(2x)$ , for  $0 \leq x \leq 2\pi$ .

- (a) Describe the transformations which have been applied to the graph of  $y = \cos x$  to obtain  $f(x)$ . (3 marks)

- horizontal dilation scale factor  $\frac{1}{2}$
- vertical dilation scale factor (-3)  $\begin{cases} \rightarrow \text{vertical dilation s.f. 3} \\ \rightarrow \text{reflection about } x \text{ axis} \end{cases}$
- vertical translation 5 units up

Behaviours: must have all 3 compound transformations ✓ protocol order ✓

- (b) Determine the coordinates of the minimum point(s) of  $f(x)$ . (2 marks)

$$\min \cos x = -1 \quad \therefore \min 5 - 3 \cos(2x) = 2.$$

Co-ordinates of minimum points =  $(0, 2)$   $(\pi, 2)$   $(2\pi, 2)$

Behaviours: All three points ✓ (working not required)

- (c) [Only one point with working ✓] Describe the phase shift required to express  $f(x)$  as a function of sine resulting in the same graph over the given domain. (1 mark)

Horizontal translation of  $\frac{\pi}{4}$  to the right ✓

Behaviours: correct solution

Note: Equation  $y = -3 \sin(2x - \frac{\pi}{2}) + 5$  (not required.)

$$\text{or } y = -3 \sin\left[2\left(x + \frac{\pi}{4}\right)\right] + 5$$

See next page

## Question 2

(3 marks)

Determine the centre and radius of the circle given by the equation:

$$x^2 + y^2 + 6x - 10y = 2.$$

$$x^2 + 6x + 9 - 9 + y^2 - 10y + 25 - 25 = 2.$$

$$(x + 3)^2 + (y - 5)^2 - 9 - 25 = 2.$$

$$(x + 3)^2 + (y - 5)^2 = 36.$$

Hence circle centre =  $(-3, 5)$ 

radius = 6.

Behaviours : • completes the square ✓  
(working)

• states solution in form ✓

$$(x - h)^2 + (y - k)^2 = r^2$$

• states correct radius ✓

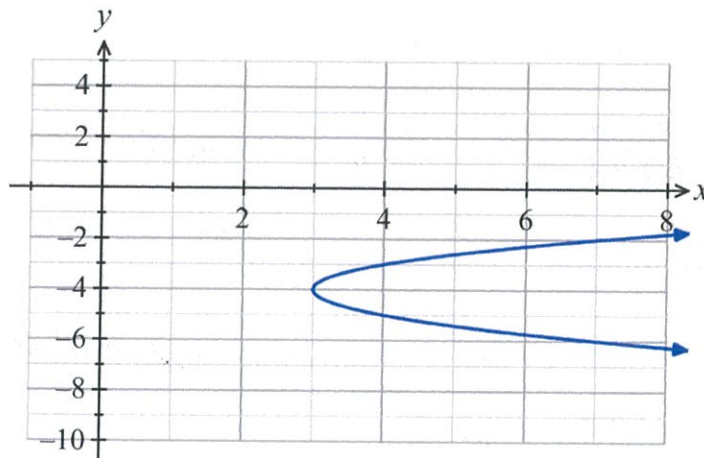
• states correct circle centre ✓

See next page

Question 3

(5 marks)

A graph has been drawn below.



Determine:

(a) with reason(s), if the graph is a relation or function.

(2 marks)

- relation ✓
- fails vertical line test ✓

(b) the equation of the axis of symmetry.

(1 mark)

$$y = -4.$$

(c) the equation which represents the graph.

(2 marks)

$$(y + 4)^2 = x - 3.$$

$$(or) y = \pm \sqrt{x - 3} - 4$$

$$(or) (y + 4)^2 + 3 = x.$$

Behaviour : recognises { vertical translation (4 units down) ✓  
 : horizontal translation (3 units right) ✓  
 : final solution statement ✓

See next page

Question 4

(8 marks)

Solve:

(a)  $(2 \cos x + 1)(\sin x - 2) = 0$  for  $0 \leq x \leq 3\pi$ .

(4 marks)

N.F.L. either  $2 \cos x + 1 = 0$  or  $\sin x - 2 = 0$  } ✓

$\therefore \cos x = -1/2$ .

$\sin x = 2$

no solution ✓

$x = 2\pi/3, 4\pi/3, 8\pi/3$  ✓

Behaviour: NFL ✓

: no solution  $\sin x = 2$  ✓

: all three solutions ✓

(b)  $4(2 \sin^2 x + \cos^2 x) - 6 = 0$  for  $0 \leq x \leq 2\pi$ .

(4 marks)

Either  $2 \sin^2 x + \cos^2 x = 3/2$

or  $2(1 - \cos^2 x) + \cos^2 x = 3/2$ .

$\therefore 2 \sin^2 x + 1 - 2 \sin^2 x = 3/2$ .

$2 - 2 \cos^2 x + \cos^2 x = 3/2$ .

$\sin^2 x = 3/2 - 1$

$2 - 1.5 = \cos^2 x$ .

$\sin^2 x = 1/2$ .

$\therefore \cos x = \pm 1/\sqrt{2}$ .

$\sin x = \pm 1/\sqrt{2}$

Hence  $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ .

Behaviours:

✓ 1. substitutes trig identity

✓ 2. reduces to  $\sin x = \pm 1/\sqrt{2}$ .

✓ 3. all 4 solutions. (2)

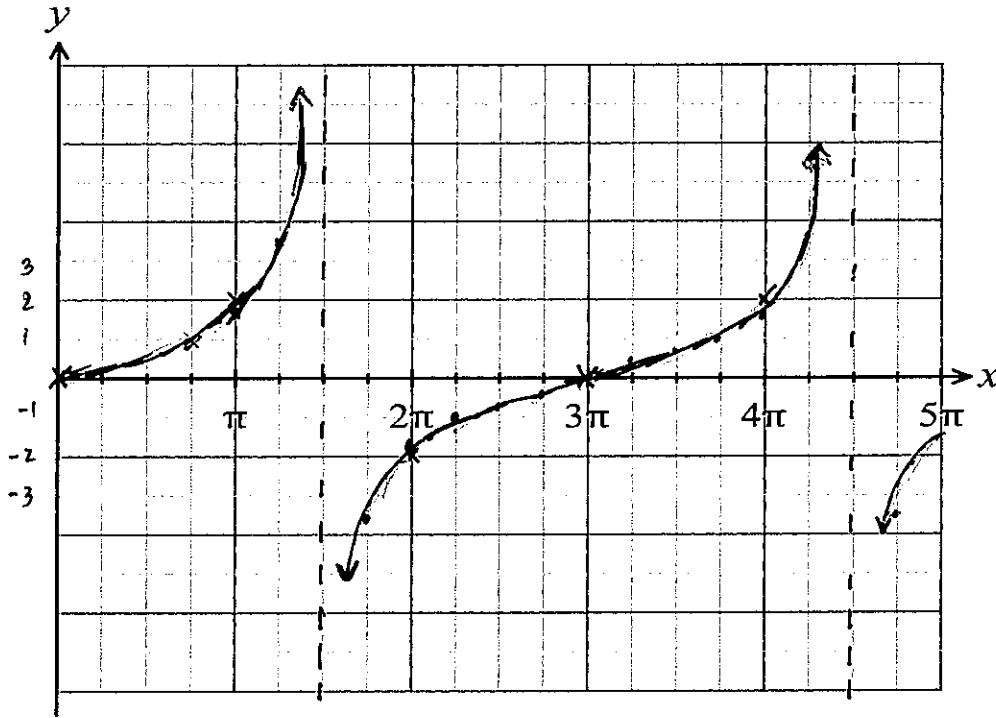
• only 2 solutions (1)

See next page

Question 5

(7 marks)

- (a) On the axes below, sketch the graph of  $y = \tan\left(\frac{x}{3}\right)$  over the interval  $0 \leq x \leq 5\pi$ , clearly indicating the equations of any asymptotes. (3 marks)



Behaviours

- ✓ • correct asymptotes and period
- ✓ • correct intercepts
- ✓ • correct shapes.

- (b) Solve the following equations over the interval  $0 \leq x \leq 5\pi$ , giving exact answers,

(i)  $\tan\left(\frac{x}{3}\right) = -1$ .

$\tan y = -1$

(2 marks)

$\therefore y = \frac{3\pi}{4}$

Behaviours: Structure ✓

: solution ✓

$\frac{x}{3} = \frac{3\pi}{4}$

$\therefore x = \frac{9\pi}{4}$

(ii)  $\tan\left(\frac{x}{3}\right) - \sqrt{3} = 0$ .

(2 marks)

$\tan \frac{x}{3} = \sqrt{3}$

$\tan y = \sqrt{3}$

Behaviours:

structure ✓

$y = \frac{\pi}{3}$

both solutions ✓

$x = \pi, 4\pi$

$\therefore \frac{x}{3} = \frac{\pi}{3}$

$\therefore x = \pi$

End of questions

**Note:** no penalty (i)  
(ii)  
if correct solutions  
(no working)

**Additional working space**

Question number: \_\_\_\_\_



**Additional working space**

Question number: \_\_\_\_\_





## MATHEMATICS METHODS Year 11

### Section Two:

### Calculator-assumed

Your name \_\_\_\_\_

Teacher name \_\_\_\_\_

### Time and marks available for this section

Reading time before commencing work: 2 minutes  
Working time for this section: 15 minutes  
Marks available: 15 marks

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet (retained from Section One)

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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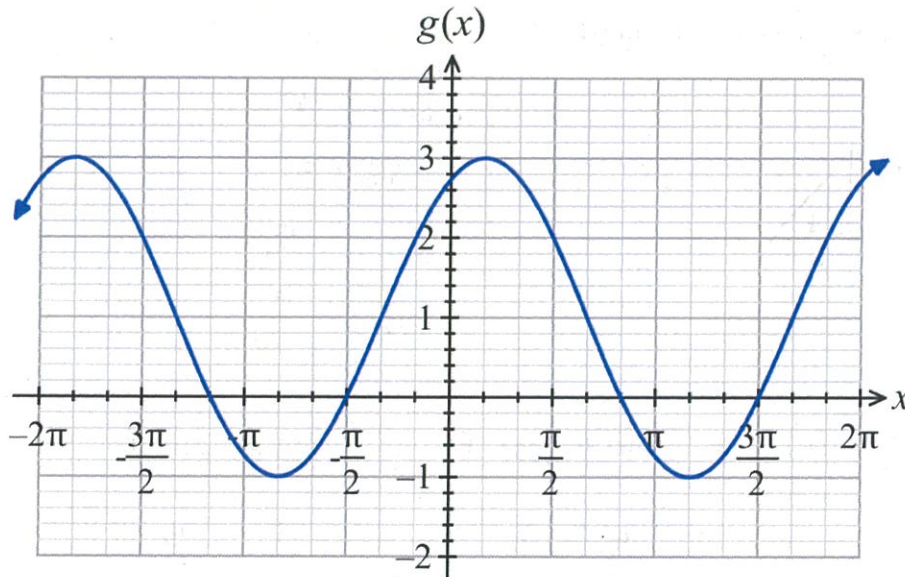
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Question 6

(3 marks)

Determine the equation of the trigonometric function,  $g(x)$ , shown below in terms of a sine function.



amplitude = 2

period =  $2\pi$

vertical translation = 1

horizontal translation =  $\frac{\pi}{3}$ . (Left)

$$g(x) = 2 \sin\left(x + \frac{\pi}{3}\right) + 1.$$

Behaviours: • amplitude

• phase shift

• centre line

(2 marks)

Question 7

The length of a string in musical instruments varies inversely to the frequency of the vibrations. This generates the sound or pitch of the note.

If an 11-inch string has a frequency of 400 cycles per second, then determine the frequency of a bass guitar which uses 30-inch strings.

let  $f$  = frequency

$l$  = length

$$f = \frac{k}{l}$$

$$\therefore k = f \times l$$

$$= 400 \times 11$$

$$= 4400$$

$$\therefore f_2 = \frac{4400}{30}$$

$$= 146\frac{2}{3}$$

cycles per second.

Note: no penalty for rounding

See next page

Behaviours

• calculates  $k$  ✓

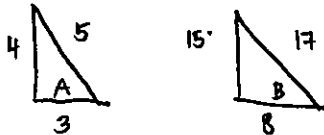
• calculates  $f_2$  ✓

## Question 8

(4 marks)

$A$  and  $B$  are both acute angles with  $\cos(B) = \frac{8}{17}$  and  $\sin(A) = \frac{4}{5}$ .

Determine the exact value of  $\sin(A + B)$ .



$$\cdot \sin A = \frac{4}{5}$$

$$\cdot \cos A = \frac{3}{5}$$

$$\cdot \sin B = \frac{15}{17}$$

$$\cdot \cos B = \frac{8}{17}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

$$= \frac{4}{5} \times \frac{8}{17} + \frac{3}{5} \times \frac{15}{17}.$$

$$= \frac{32}{85} + \frac{45}{85}$$

$$= \frac{77}{85}$$

Behaviours : calculates  $\cos A$  ✓  
 : calculates  $\sin B$  ✓

: uses correct substitution ✓

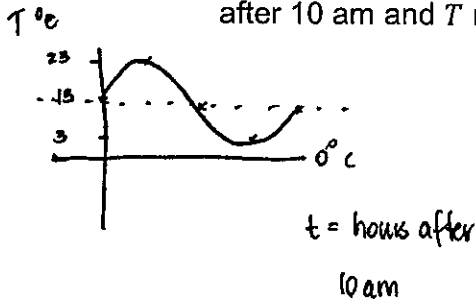
: final solution ✓

Question 9

(5 marks)

In a particular city in South Africa it is known that the outdoor temperature ranges from a low of  $3^{\circ}\text{C}$  to a high of  $23^{\circ}\text{C}$  each day. Scientists measure the outdoor temperature throughout the day and discover that it can be modelled by a sine function. The temperature midpoint occurs at 10 am and at 10 pm with the temperature peaking in the afternoon.

- (a) Determine an equation of this function where  $t$  represents the time, in hours, after 10 am and  $T$  represents the outdoor temperature in  $^{\circ}\text{C}$ . (3 marks)



$$T = 10 \sin\left(\frac{\pi}{12} t\right) + 13$$

- Behaviours :
- : centreline ✓
  - : vertical dilat ✓
  - : period + b ✓
  - : final equation

- (b) When the outdoor temperature falls below  $10^{\circ}\text{C}$  heating systems are used. Determine the times, to the nearest minute, at which heating systems are in use. (3 marks)

$$\text{let } T^{\circ} = 10^{\circ}\text{C}$$

$$\begin{aligned}
 t &= 13.164 \\
 \text{Hours after 10am.} &= 22.836
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{hours after 10am}$$

✓ calculation of both 't' values

$$\therefore 23.16 = 11:10\text{pm}$$

$$32.84 = 8:50\text{am.}$$

Hence

✓ converts 't' into time

$$\begin{array}{ccccccc}
 11:10\text{pm} & < & t & < & 8:50\text{am.} \\
 \uparrow & & & & \uparrow \\
 \text{on} & & & & \text{off.}
 \end{array}$$

✓ lists both boundaries to nearest minute

End of questions

**CALCULATOR-ASSUMED**

**6 MATHEMATICS METHODS Year 11**

**Additional working space**

Question number: \_\_\_\_\_



**Additional working space**

Question number: \_\_\_\_\_

